Math 1A
Midterm 2 Review Answers

[1] $\csc 0.5 \approx \csc\frac{\pi}{6} + \left(-\csc\frac{\pi}{6}\cot\frac{\pi}{6}\right)\left(0.5 - \frac{\pi}{6}\right) = \frac{6+(\pi-3)\sqrt{3}}{3}$

[2] $dx = \Delta x = 0.5$

$\Delta y = y(2+0.5) - y(2) = \frac{4}{25} - \frac{1}{4} = -\frac{9}{100}$

$dy = \left.\frac{dy}{dx}\right|_{x=2} \Delta x = -\frac{2}{(2)^3}(0.5) = -\frac{1}{8}$

[3] $\frac{d^3}{dx^3}\sec x$

$= \frac{d^2}{dx^2}\sec x \tan x$

$= \frac{d}{dx}(\sec x \tan^2 x + \sec^3 x)$

$= \sec x \tan^3 x + 2\sec^3 x \tan x + 3\sec^3 x \tan x$

$= \sec x \tan^3 x + 5\sec^3 x \tan x$

[4] **SIMPLIFY THE FUNCTION BY DIVISION BEFORE TAKING DERIVATIVES** $s(t) = 2t^{\frac{5}{2}} + 4t^{\frac{3}{2}} - 3t^{-\frac{1}{2}}$

[a] $s'(t) = 5t^{\frac{3}{2}} + 6t^{\frac{1}{2}} + \frac{3}{2}t^{-\frac{3}{2}} \Rightarrow s'(1) = \frac{25}{2}$

[b] $s''(t) = \frac{15}{2}t^{\frac{1}{2}} + 3t^{-\frac{1}{2}} - \frac{9}{4}t^{-\frac{5}{2}} = \frac{3}{4}t^{-\frac{5}{2}}(10t^3 + 4t^2 - 3)$

[5] The given line has slope $-\frac{1}{12}$, so the tangent lines must have slope 12

$\frac{dy}{dx} = 3x^2 = 12 \Rightarrow x = \pm 2$

The tangent lines are $y - 9 = 12(x-2)$ and $y + 7 = 12(x+2)$

[6] Since the function is quadratic, $f(x) = ax^2 + bx + c$ where $f(1) = -1$, $f'(1) = 3$ and $f(2) = 4$

So, $a + b + c = -1$, $2a + b = 3$ and $4a + 2b + c = 4$

Solving for the coefficients, $f(x) = 2x^2 - x - 2$

The tangent line is $y - 4 = 7(x-2)$

[7] $f'(x) = \frac{3x^2(1+x^2) - x^3(2x)}{(1+x^2)^2} = \frac{2x^2 + x^4}{(1+x^2)^2}$

$f''(x) = \frac{(4x+4x^3)(1+x^2)^2 - (2x^2+x^4)(2(1+x^2)(2x))}{(1+x^2)^4}$

$f''(1) = \frac{(4+4)(1+1)^2 - (2+1)(2(1+1)(2))}{(1+1)^4} = \frac{32-24}{16} = \frac{1}{2}$ **NO NEED TO SIMPLIFY $f''(x)$ to find $f''(1)$**

[8] [a] $k(2) = 8f(2)$ and $k'(x) = 3x^2 f(x) + x^3 f'(x) \Rightarrow k'(2) = 12f(2) + 8f'(2)$

The tangent line is $y + 8 = 12(x - 2)$

[b] $j(-1) = \dfrac{1}{f(-1)}$ and $j(x) = \dfrac{2xf(x) - x^2 f'(x)}{[f(x)]^2} \Rightarrow j(-1) = \dfrac{-2f(-1) - f'(-1)}{[f(-1)]^2}$

The tangent line is $y - \dfrac{1}{2} = -\dfrac{1}{4}(x + 1)$

[c] $m(-3) = \tan^{-1}(g(-3))$ and $m'(x) = \dfrac{1}{1 + [g(x)]^2} g'(x) \Rightarrow m'(-3) = \dfrac{1}{1 + [g(-3)]^2} g'(-3)$

The tangent line is $y + \dfrac{\pi}{4} = x + 3$

[d] $n(4) = g(f(4))$ and $n'(x) = g'(f(x))f'(x) \Rightarrow n'(4) = g'(f(4))f'(4) = g'(3)f'(4)$

The tangent line is $y = -3(x - 4)$

[9] **ANSWER WITHHELD (SHOW ME YOUR SOLUTION FOR VERIFICATION)**

[10] $f'(x) = 4 - 3\sec^2 x = 0 \Rightarrow \sec x = \pm\dfrac{2}{\sqrt{3}} \Rightarrow \cos x = \pm\dfrac{\sqrt{3}}{2} \Rightarrow x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

[11] $f'(x) = g(x^2) + xg'(x^2)(2x) = g(x^2) + 2x^2 g'(x^2)$

$f''(x^2) = g'(x^2)(2x) + 4xg'(x^2) + 2x^2 g''(x^2)(2x) = 6xg'(x^2) + 4x^3 g''(x^2)$

[12] $5(1 + x^2y^3)^4(2xy^3 + 3x^2y^2\dfrac{dy}{dx}) = 4x^3e^y + x^4e^y\dfrac{dy}{dx}$

$5(1+0)^4(0+0) = 4(-1)(1) + (1)(1)\left.\dfrac{dy}{dx}\right|_{(-1,\,0)} \Rightarrow \left.\dfrac{dy}{dx}\right|_{(-1,\,0)} = 4$ **NO NEED TO SIMPLIFY dy/dx to find $dy/dx|_{(-1,\,0)}$**

$y = 4(x + 1)$

[13] $y = ax^4 \Rightarrow \dfrac{dy}{dx} = 4ax^3$

$x^2 + 4y^2 = b^2 \Rightarrow 2x + 8y\dfrac{dy}{dx} = 0 \Rightarrow \dfrac{dy}{dx} = -\dfrac{x}{4y}$

$4ax^3\left(-\dfrac{x}{4y}\right) = -\dfrac{ax^4}{y} = -\dfrac{y}{y} = -1$

[14] $\ln y = \dfrac{\ln \sin x}{x}$

$\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{\ln \sin x - x\dfrac{1}{\sin x}\cos x}{x^2} = \dfrac{\ln \sin x - x\cot x}{x^2}$

$\dfrac{dy}{dx} = \dfrac{\ln \sin x - x\cot x}{x^2}(\sin x)^{\frac{1}{x}}$

[15] $f(x) = xe^{-x}$, $a = -1$ and $\lim\limits_{h \to 0}\dfrac{(h-1)e^{1-h} + e}{h} = f'(-1) = (e^{-x} - xe^{-x})\Big|_{x=-1} = 2e$